Realising Energy-Aware Communication over Fading Channels under QoS Constraints

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***Abstract*—There exists a trade-off between energy consumption and spectral efficiency in wireless communication systems under quality of service (QoS) constraints. This paper studies the use of effective capacity theory to characterise the maximum supported channel capacity over fading channels whilst considering both QoS constraints and energy consumption. Moreover, a generalised fading channel model, i.e., the hyper Fox's *H* fading model, is considered that includes many practical fading channel models as special cases, e.g., Rayleigh, Rician, Weibull and Nakagami-m fading channel models. The results are readily applicable to design energy-aware communication systems over fading channels with QoS constraints, e.g., wireless sensor networks and smart grid communication systems.**

I. Introduction

Compared with wired communications, wireless communications have a more flexible and scalable architecture. However, almost all wireless communication networks have to balance two important aspects, i.e., conserving the limited system resources and providing reliable services. In practice, the energy consumption and available frequency bandwidth are always confined due to the cost and interference regulation. According to Shannon capacity theory, in order to improve the throughput under the same channel condition, it has to increase the transmit power or the frequency bandwidth. Hence there exists a trade-off between spectral efficiency and energy consumption. Unfortunately, the traditional Shannon capacity theory [1] cannot take delay aspects into account to support the communication system design in real-time applications, e.g., smart grid applications (virtual power plant and distribution automation) that shall consider the time delay introduced by communication systems.

Wireless communication is not as reliable as its wired counterpart, since the wireless channel varies randomly with time. This fact contributes to the uncertainty in providing QoS guarantee, such as guaranteeing the time delay and providing a minimum throughput. In order to address this issue, the theory of effective capacity was proposed in [2] that studies the maximum supported bit rate over a fading channel under statistical delay constraint. Using the effective capacity theory, the general relation between bit energy consumption and spectral efficiency with QoS constraints was studied [3]. In [1], the general trade-off among energy, bandwidth and delay was considered, where Rayleigh fading channels were considered. In [4], the problem of maximum the effective capacity with optimum bit energy over Nakagami-m fading channels was analysed. However, most existing work built on Rayleigh or Nakagami-m fading channels, whereas a general quantified analysis over practical fading channels has been insufficiently studied.

In this paper, we consider the communication systems' energy performance under statistical QoS constraints over hyper Fox's H fading channels, which is a generalized fading model including many practical fading models as special cases, e.g., Rayleigh, Rician, Nakagami-m, Weibull, Weibull/Gamma and Generalized K fading model. In this way, the derived quantified analysis of energy performance and spectral efficiency is readily applicable to the analysis of the aforementioned fading channels, which can be used in designing energy-aware communications.

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II. System Model

A. Channel Model

In this paper, a point-to-point communication system model similar to the one studied in [1] and [4] is considered, which is shown in Fig. 1. We assume that only the receiver has perfect channel state information (CSI) and the channels are flat block fading. Thus the channel input-output relation is given by [4]

$$y[j] = h[j]x[j] + n[j] \quad \text{for} \quad j = 1, 2, \dots \tag{1}$$

where $x[j]$ and $y[j]$ are the channel input and output, and $n[j]$ is the zero-mean, circularly-symmetric, complex Gaussian noise with variance N_0. Also $h[j]$ denotes the channel fading coefficient. The SNR ρ is defined by $\rho = \frac{P}{BN_0}$, where P is the average transmit power and B is the bandwidth. Further, the instantaneous channel gain is denoted by $\gamma[j] = |h[j]|^2$ and the average channel gain is denoted by $\Omega = \mathbb{E}\{|h[j]|^2\}$. For notational brevity, the time index j is omitted.

B. Hyper Fox's H fading channel

Hyper Fox's H fading model is a generalized fading model [5], which can exact represent or approximate various practical fading models as special cases, such as Rayleigh, Rician, Weibull, Nakagami-m, Lognormal, Generalized K, Weibull/Gamma and standard Fox's H fading model. Let γ be a random variable following the hyper Fox's H fading

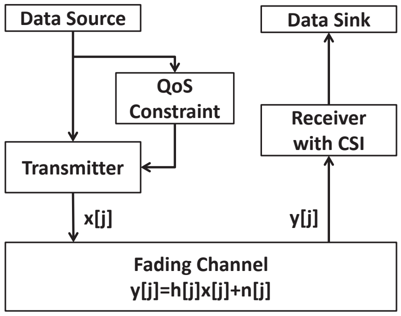

Fig. 1. System model

distribution, then its PDF is given by [5]

$$f(\gamma) = \sum_{k=1}^{K} H[\gamma, \mathbf{O}_k, \mathbf{P}_k], \tag{2}$$

where

$$\begin{cases} \mathbf{O}_k = (m_k, n_k, p_k, q_k), \\ \mathbf{P}_k = (u_k, v_k, \mathbf{c}_k, \mathbf{d}_k, \mathbf{C}_k, \mathbf{D}_k) \end{cases} \tag{3}$$

whereas $\mathbf{c} = (c_1, \ldots, c_p), \mathbf{d} = (d_1, \ldots, d_q), \mathbf{C} = (C_1, \ldots, C_p)$ and $\mathbf{D} = (D_1, \ldots, D_q)$. The parameters K, $\mathbf{O}_k$ and $\mathbf{P}_k$ satisfy the conditions given in [5]. The Fox's H function is defined by a single Mellin-Barnes type of contour integral as [6, Ch. 1.2]

$$H\left[\gamma, \mathbf{O}, \mathbf{P}\right] = \frac{u}{2\pi i} \oint_{\mathcal{L}} \Phi\left(z\right) \left(v\gamma\right)^{-z} dz, \tag{4}$$

where $i = \sqrt{-1}$, $z \neq 0$ and $\mathcal{L}$ is a suitable contour. The function $\Phi\left(z\right)$ is defined by [6]

$$\Phi\left(z\right) = \frac{\prod_{j=1}^{m} \Gamma\left(d_j + D_j z\right) \prod_{j=1}^{n} \Gamma\left(1 - c_j - C_j z\right)}{\prod_{j=n+1}^{p} \Gamma\left(c_j + C_j z\right) \prod_{j=m+1}^{q} \Gamma\left(1 - d_j - D_j z\right)}. \tag{5}$$

Further, we define the following function

$$\Psi(z) = u v^{-z} \Phi\left(z\right), \tag{6}$$

which will be used to deduce the spectral efficiency metric in the following part.

C. Effective capacity theory

Effective rate is the maximum constant rate that a fading channel can support under statistical delay constraints. The effective capacity of the service process can be written as [2]

$$\alpha(\rho, \theta) = -\frac{1}{\theta T} \ln \mathbb{E}\left\{e^{-\theta T C}\right\}, \quad \theta \neq 0, \tag{7}$$

where C represents the system's throughput during a single time block and T denotes the duration of a time block. The QoS exponent θ is given by [2]

$$\theta = -\lim_{z \to \infty} \frac{\ln \Pr\{L > z\}}{z}, \tag{8}$$

where L is the equilibrium queue-length of the buffer at the transmitter, and $\Pr\{\cdot\}$ is the probability of an event. The QoS exponent θ has to satisfy the constraint $\theta \geq \theta_0$, where θ_0 is the minimum required QoS exponent. Larger θ_0 corresponds to a tighter delay constraint, whereas $\theta_0 = 0$ implies no delay constraints. Also when $\theta_0 \to 0$, the effective capacity approaches the Shannon's channel capacity [7].

III. Effective Capacity over General Fading Channels

According to Shannon's theory, if we want to improve the spectral efficiency, i.e. transmit more bits under the same bandwidth, a direct way is to increase the SNR, which can be achieved by increase the transmit power. But in practical, increase the transmit power will usually result in higher energy consumption and more interference to other transceivers. In this section, we will quantify the relation between bit energy and spectral efficiency under a dedicated QoS constraint.

In this paper, the energy per information bit E_b/N_0 (in dB) is used as the metric for energy performance, which can provide a unified way to measure and compare the energy performance between different wireless communications systems [4]. Moreover, effective capacity normalized by bandwidth $R(\rho, \theta)$ (in bit per second per hertz) is used as the metric for spectral efficiency. Specifically, the spectral efficiency metric is defined as [1]

$$R(\rho, \theta) = \frac{\alpha(\rho, \theta)}{B} = -\frac{1}{\theta T B} \ln \mathbb{E}\{e^{-\theta T C}\}. \tag{9}$$

The effective capacity is a concave function of the SNR ρ and the minimum bit energy is achieved as $\rho \to 0$ [1], where the effective capacity can be approximated by

$$R\left(\frac{E_b}{N_0}, \theta\right) \approx S_0 \log_2\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right), \tag{10}$$

where S_0 denotes the capacity slope in bits/s/Hz/(3dB) and $E_b/N_{0\min}$ is the minimum energy per information bit required under QoS constraints, which are given by [1]

$$\begin{cases} \frac{E_b}{N_0}_{\min} = \lim_{\rho \to 0} \frac{\rho}{R\left(\rho, \theta\right)} = \frac{1}{R'\left(0, \theta\right)} \\ S_0 = -\frac{2\left[R'\left(0, \theta\right)\right]^2 \ln 2}{R''\left(0, \theta\right)} \end{cases} \tag{11}$$

where $R'\left(0, \theta\right)$ and $R''\left(0, \theta\right)$ denote the first and second order partial derivatives with respect to ρ evaluated at ρ=0.

When considering hyper Fox's H fading channels, these metrics can be calculated via the following theorem.

Theorem 1: When only the receiver has perfect CSI, the bit energy, spectral efficiency and QoS exponent can be quantified by (10), with the minimum bit energy $\frac{E_b}{N_0}_{\min}$ and capacity slope S_0 under hyper Fox's H fading channel given by

$$\begin{cases} \frac{E_b}{N_0}_{\min} = \frac{\ln 2}{\Omega} \\ S_0 = \frac{2}{\frac{(A+1)}{\Omega^2} \sum_{k=1}^{K} \Psi_k(3) - A} \end{cases} \tag{12}$$

where $A=\frac{\theta TB}{\ln 2}$ and the function $\Psi(z)$ is defined in (6).

Proof: By calculating the first and second order partial derivatives with respect to ρ using (9), we have $\frac{E_b}{N_0}_{\min}=\frac{\ln 2}{\mathbb{E}\{\gamma\}}$ and $S_0=\frac{2}{(A+1)\mathbb{E}\{\gamma^2\}/(\mathbb{E}\{\gamma\})^2-A}$ [1, eq.(18)]. Then using Mellin transform [6, eq.(2.8)] and (6), (12) can be achieved. $\square$

Remark: It can be seen that the minimum required bit energy is only related to the average channel gain and it does not depend on specific channel fading parameters, which is consistent with [1]. The capacity slope S_0 depends not only on the QoS requirement θ, but also fading parameters. In this context, the bit energy can be adjusted according to the communication requirements that realises the energy-aware communication while providing statistical QoS guarantees.

As discussed in Section II-C, when $\theta=0$, there is no delay requirement on the transmission. Hence the effective capacity becomes Shannon capacity, and the metrics can be used in traditional Shannon capacity analysis by letting $\theta=0$, which can be calculated by the following corollary.

Corollary 1: When only the receiver has perfect CSI and there is no delay constraint, the minimum bit energy and capacity slope under hyper Fox's H fading channel reduce to

$$\begin{cases} \frac{E_b}{N_0}_{\min}=\frac{\ln 2}{\Omega} \\ S_0=\frac{2\Omega^2}{\sum_{k=1}^{K}\Psi_k(3)} \end{cases} \tag{13}$$

Proof: These results can be obtained by substituting $\theta=0$ into Theorem 1. $\square$

Hyper Fox's H fading model is a general model, but its parameters have physical meaning only when specific fading model is considered. Hence in the following, we consider several typical and practical fading models, namely Rayleigh, Weibull, Nakagami-m, Weibull/Gamma and Generalized K fading channels as examples to show the usage of Theorem 1. All these fading models are special cases of hyper Fox's H fading model.

- *Rayleigh fading:* If there are only non-light-of-sight paths, measurements show that the Rayleigh distribution can characterize the statistical distribution of the received signal's envelop [8]. The instantaneous SNR parameter sequences for Rayleigh distribution are [8, Table IX] $\mathbf{O}=(1,0,0,1)$ and $\mathbf{P}=(1,1,\text{—},\frac{1}{2},\text{—},\frac{1}{2})$. By using Theorem 1, the capacity slope can be given by $S_0=\frac{2}{A+2}$.
- *Nakagami-m fading:* The Nakagami-m distribution has been first introduced in [9], which can characterize the envelop of the received signal propagating in ionospheric and tropospheric environments. The instantaneous SNR parameter sequences are [8, Table IX] $\mathbf{O}=(1,0,0,1)$ and $\mathbf{P}=(\frac{m}{\Gamma(m)},m,\text{—},m-1,\text{—},1)$, where m describes the fading severity. The capacity slope can be given by $S_0=\frac{2m}{A+m+1}$. The Nakagami-m fading includes the Rayleigh fading ($m=1$) as special cases and it can approximate Nakagami-q (or Hoyt) fading ($m<1$) and Rician fading ($m>1$).
- *Weibull fading:* Weibull fading model can characterize the effect of clusters of multipath waves propagating in non-homogeneous environment [7]. The instantaneous SNR parameter sequences are [8, Table IX] $\mathbf{O}=(1,0,0,1)$ and $\mathbf{P}=(\Gamma(1+2/\beta),\Gamma(1+2/\beta),\text{—},1-2/\beta,\text{—},1-2/\beta)$, where β describes fading severity. The capacity slope can be given by $S_0=\frac{2}{\frac{\Gamma(4/\beta+1)(A+1)}{\Gamma^2(2/\beta+1)}-A}$. This model includes Rayleigh fading ($\beta=2$) and exponential fading ($\beta=1$) as special cases.
- *Weibull/Gamma fading:* Weibull/Gamma fading model characterizes the composite effect of Weibull multipath fading and gamma shadowing [10]. The instantaneous SNR parameter sequences for Weibull/Gamma fading model are [8, Table IX] $\mathbf{O}=(2,0,0,2)$ and $\mathbf{P}=(\frac{\Gamma(1+2/\beta)}{\psi\Gamma(1/\psi)},\frac{\Gamma(1+2/\beta)}{\psi},\text{–},(\frac{1}{\psi}-1,1-\frac{2}{\beta}),\text{–},(1,\frac{2}{\beta}))$, where the fading severity parameter $\beta>0$ and shadowing figure $\psi\geq 0$. The capacity slope can be given by $S_0=\frac{2}{\frac{\psi\Gamma(4/\beta+1)(1/\psi+1)(A+1)}{\Gamma^2(2/\beta+1)}-A}$. This model reduces to Weibull fading model when $\psi=0$.
- *Generalized K-fading:* Generalized K-fading model has been first introduced in [11] to model the intensity of radiatio scattered with a non-uniform phase distribution, which accounts for the composite effect of Nakagami-m multipath fading and gamma shadowing. The instantaneous SNR parameter sequences for generalized K-fading model are [8, Table IX] $\mathbf{O}=(2,0,0,2)$ and $\mathbf{P}=(\frac{m}{\psi\Gamma(m)\Gamma(1/\psi)},\frac{m}{\psi},\text{–},(m-1,\frac{1}{\psi}-1),\text{–},(1,1))$, where multipath fading severity parameter $m\geq\frac{1}{2}$ and shadowing figure $\psi\in[0,2]$. The capacity slope can be given by $S_0=\frac{2}{\frac{(m+1)(1/\psi+1)}{m/\psi}(A+1)-A}$. Specially, when $\psi=0$, the generalized K-fading reduces to the Nakagami-m fading.

Here we list the capacity slope S_0 of the aforementioned typcial fading channels in Table I. It can be verified that when $\theta=0$, these results coincide with [8].

TABLE I
CAPACITY SLOPE S_0 FOR TYPICAL FADING CHANNELS

Fading Channel	S_0
Rayleigh	$\frac{2}{A+2}$
Nakagami-m	$\frac{2m}{A+m+1}$
Weibull	$\frac{2}{\frac{\Gamma(4/\beta+1)(A+1)}{\Gamma^2(2/\beta+1)}-A}$
Weibull/Gamma	$\frac{2}{\frac{\psi\Gamma(4/\beta+1)(1/\psi+1)(A+1)}{\Gamma^2(2/\beta+1)}-A}$
Generalized K	$\frac{2}{\frac{(m+1)(1/\psi+1)}{m/\psi}(A+1)-A}$

IV. NUMERICAL RESULTS

In order to verify the the quantified relation among energy performance, spectral efficiency and QoS exponent, simulations under Weibull/Gamma fading channels have been carried out. The spectral efficiency is approximated using (10), where the parameters are calculated using Table I. Without loss of generality, the duration of a time block $T=1$ ms and the bandwidth of the system $B = 1$ kHz are assumed. For each simulated scenario, we run 10^7 trails with unit power.

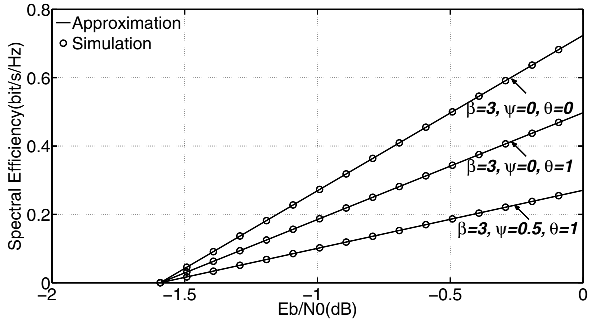

Fig. 2. Spectral efficiency versus energy performance over Weibull/Gamma fading channels.

It can be seen that the proposed approximation fits tight with the simulation resutls in Fig. 2. The results in the case $\beta = 3, \psi = 0, \theta = 0$ denote the Shannon capacity under Weibull fading channels, which agree with the results in [8] and support the validation of our derivation. With the same

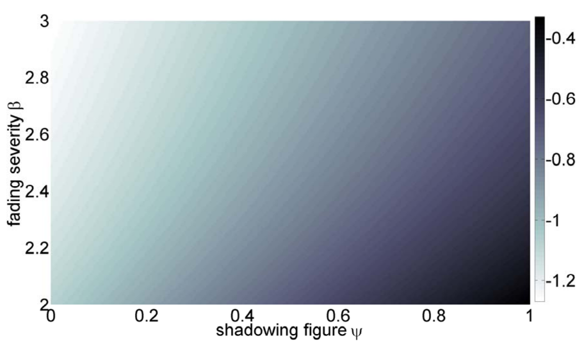

Fig. 3. Required bit energy for different fading severity β and shadowing figure ψ in Weibull/Gamma fading channels (Spectral efficiency 0.1 bit/s/Hz,θ=1)

spectral efficiency and QoS constraint, the bit energy under different fading severity and shadowing figure is shown in Fig. 3. The increase of multipath fading is indicated by the decrease of fading severity β, while the shadowing gets sever indicated by the increase of the shadowing figure ψ. It can be seen from Fig. 3, in order to fight against the fading effect caused by multipath fading and shadowing, the energy required to maintain the same spectral efficiency is increased in an exponential way.

It gives a more general view of the effect of QoS requirement θ and energy efficient metric E_b/N_0 on the spectral efficiency in Fig. 4. It is indicated from both Fig. 2 and Fig. 4 that under the same bit energy, the spectral efficiency decreases as the delay requirement becomes more strigent, which is corresponding to the increase of QoS requirement θ. When the fading parameters are known as a prior, the quantified relation as proposed in Theorem 1 can be used in the study of trade-offs between spectral efficiency and energy performance.

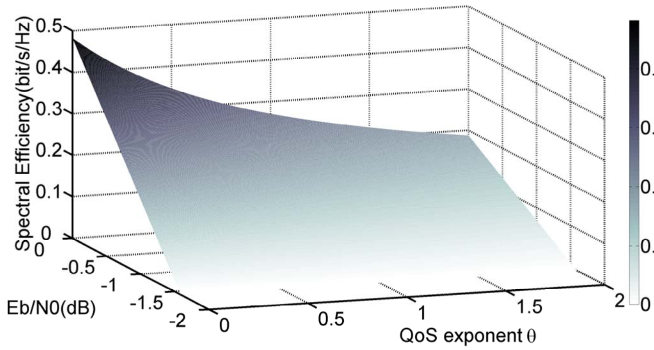

Fig. 4. Spectral efficiency v.s. bit energy E_b/N_0 and QoS exponent θ in Weibull/Gamma fading channels (β=3,ψ=0.5)

V. Conclusion

In this paper, the relationship between bit energy and spectral efficiency has been studied considering the hyper H fading channel and QoS requirements. The results have shown that the proposed approach is readily applicable to various fading channels. The bit energy consumption decreases as the QoS requirement increases. It is noteworthy highlighting that by using the quantified relationship among bit energy, spectral efficiency and QoS requirement, we can adjust the energy consumption of a wireless communication system according to the QoS requirement and channel condition. This result can be used as an important foundation to realise energy-aware communications in real-time applications over fading channels with QoS constraints.

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